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Fatigue reliability analysis of composites based on residual strength

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Abstract—The fatigue reliability of Gr/PEEK composites has been investigated in the present study. In the experimental investigation, constant amplitude loading, multiple-level loading and random loading have been considered step by step during the fatigue tests. In particular, a residual strength degradation model was presented for predicting the fatigue life of composites subjected to constant amplitude fatigue loading. The accumulation of fatigue damage was determined, using the residual strength degradation model. The residual strength distribution after an arbitrary fatigue cycle can be represented by a two-parameter Weibull distribution function. A fatigue reliability analysis method for arbitrary load history was thus developed through the Weibull distribution function. Good correlation between theory and experiment was obtained for constant amplitude loading, two-level loading, three-level loading and random loading. The result of the present study can be used as a basis for the reliability design of mechanical components made of Gr/PEEK composites.

Keywords: Composite; fatigue; reliability; residual strength.

1. INTRODUCTION

Composite materials show excellent performance in their applications in many engineering fields, for example in aero industry, automobile industry, and manufacturing components of electronic products and architectural materials [1]. Many important engineering structures have been constructed using composite materials and, as such, were subjected to cyclic stress; thus, when such structures were analyzed and designed, the factor of fatigue failure had to be considered. Therefore, predicting the fatigue life and avoiding fatigue failure are critical performance parameters.

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In general, the three accepted fatigue modeling approaches for composite materials are fatigue life models, phenomenological residual strength and stiffness models, and mechanistic models [2]. The fatigue life models use *S–N* curves but do not take into account the actual damage mechanisms. The mechanistic models use damage variables related to measurable manifestations of damage mechanisms. However, the fatigue damage of composites is a complex process that involves many different damage mechanisms or modes, such as matrix cracking, fiber–matrix debonding, fiber breakage and delamination [3]. More than one of these damage mechanisms usually happen at the same time and interact with each other [4]. Theoretically, it is very difficult to construct a mechanistic model including all the damage modes. Hence, the phenomenological approach has frequently been used in order to simplify the analysis of composites.

From the phenomenological viewpoint, the damage in composites can be evaluated by measuring the changes in material properties. On the microscopic scale, the composite materials are invariably deteriorated and weakened due to the appearance, accumulation and propagation of internal damage mechanisms. On the macroscopic scale, the residual strength and stiffness is a measure for the fatigue damage. The phenomenological residual strength models use the strength degradation as a measure. By increasing the number of cycles, under cyclic stress, the fatigue strength decreases. Finally, fatigue failure takes place when the residual strength decreases to the maximum cyclic applied stress. It is an innate fatigue failure criterion. Based on these considerations, the residual strength approach was chosen as the framework of this study.

To predict the strength degradation of a composite laminate under fatigue loading, a number of residual strength models have been proposed. At first, Halpin *et al.* [5] proposed a ‘wear-out model’. Then, Hahn and Kim [6] assumed that the rate of residual strength reduction was inversely proportional to the residual strength. Reifsnider [7] suggested the rate of residual strength reduction was the power function of fatigue cycles. Rotem [8] developed a residual strength theory which was based on a cumulative damage theory. Revuelta *et al.* [9], Whitworth [10], Van Paepegem *et al.* [2] and Tserpes *et al.* [11] have made later studies in this field.

On the other hand, the mechanical properties of heterogeneous composite materials will show more varieties than those of traditionally homogeneous metallic materials, and hence the complexity of analyzing fatigue in composite materials will be harder than in traditional materials. We can reasonably employ the random theory of probability and statistics to help us explain the variances of mechanical properties in composite materials. The representative statistically based model was proposed by Yang and his co-workers [12–14]. In another example, Radhakrishnan [15] researched the correlation between statistical distributions of the fatigue life and the residual strength. Diao *et al.* [16, 17] developed the stress redistribution function to predict residual strength and fatigue life. Yao *et al.* [18] described the residual strength degradation through statistical analysis. Most researchers introduce the Weibull distribution in the residual strength model, for example, Shan *et al.* [19] and Birgoren *et al.* [20].

Composites possess a large scatter in their fatigue properties. Therefore, it is necessary to describe the fatigue strength and fatigue life of composite laminates using the theory of probability and statistics. In this paper, a statistical approach is employed to study the fatigue reliability of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ composite laminates subjected to constant, multiple-level and random cyclic loading. The statistical distribution functions of static strength, residual strength and fatigue life are obtained by fitting the experimental data to a two-parameter Weibull distribution. The Weibull distribution, which is defined by 'shape' and 'scale' parameters, is often used to model the fatigue behavior of composites.

2. EXPERIMENTAL

The testing specimens were cut from a batch of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ (APC-2, made by I.C.I. UK) composite laminates. They have been prepared by a standard procedure. The laminate specimens (Fig. 1) had the dimensions 200 mm in length, 20 mm in width, and 1.8 mm in thickness. According to the ASTM-D3039 standard, we used the MTS 810 machine (Fig. 2) to perform the unidirectional tensile tests. The displacement-controlled load was applied at an extension rate of 2 mm/min. According to the ASTM-D3479 standard, we used the same machine to perform the fatigue tests. The stress ratio is 0.1, and the frequency is 10 Hz, with loading control mode to carry out the constant amplitude loading, two-level loading, three-level loading and random loading tests.

3. ANALYTICAL MODELS

3.1. Fatigue reliability of constant amplitude loading

In present study, a statistical model, which is based on residual strength degradation, is employed to reduce the phenomenological characterization of the fatigue damage

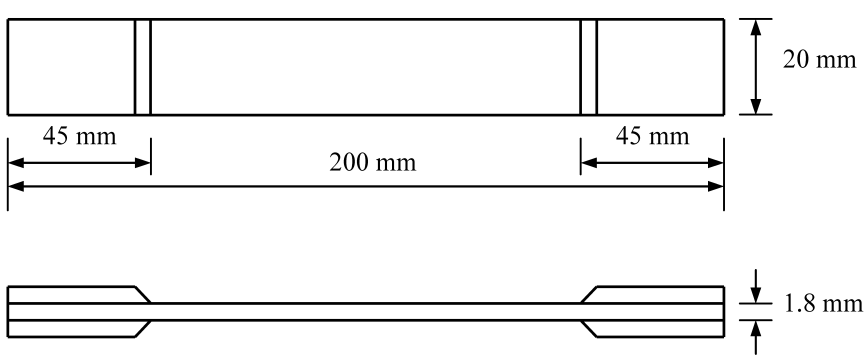

Figure 1. The dimension of the laminates specimens.

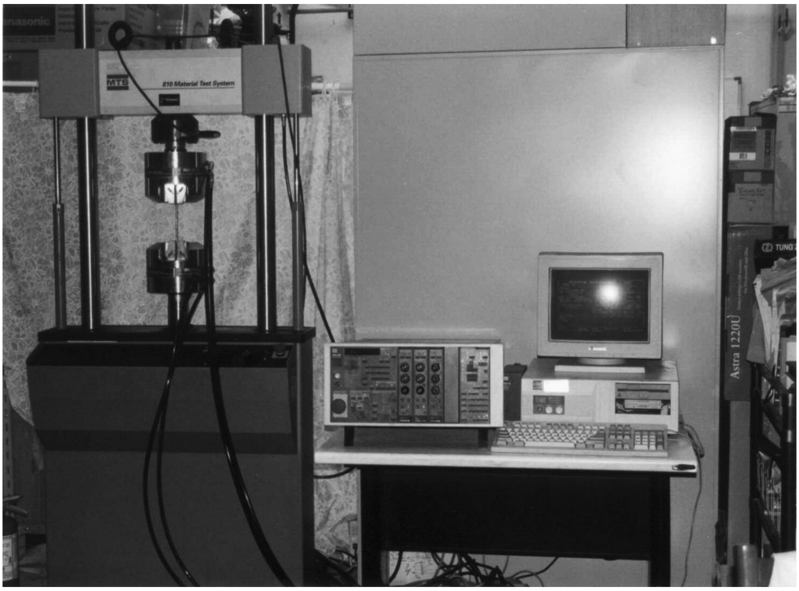

Figure 2. MTS 810 machine.

behavior of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ composite laminates. The fatigue reliability evaluation approach is based on two assumptions: (i) The static strength follows a two-parameter Weibull distribution; (ii) Residual strength after n cycles of cyclic load is related to the initial static strength through a deterministic equation.

When a composite laminate was subjected to a constant amplitude loading, the residual strength decreased when the fatigue cyclic period increased. According to previous work [7, 13, 21], the form of the degradation curve for residual strength is

$$R(n) = R(0) - [R(0) - S]\left(\frac{n}{N}\right)^{\lambda}, \tag{1}$$

where $R(0)$ is the static strength, $R(n)$ is the residual strength after n cycles tested, n is the fatigue cyclic period, N is the fatigue life, S is the fatigue stress and λ is a degradation parameter of residual strength.

Assuming the residual strength after n cycles tested is represented by a two-parameter Weibull distribution function, we can express the fatigue reliability function in the form [22]:

$$\mathcal{R}_{\mathbf{R}(n)}[S] = \mathcal{P}[\mathbf{R}(n) > S] = \exp\left[-\left(\frac{S}{\theta_{R(n)}}\right)^{\beta_{R(n)}}\right], \tag{2}$$

where $\theta_{R(n)}$ is the scale parameter of residual strength and $\beta_{R(n)}$ is the shape parameter of residual strength. The values of these two parameters can be obtained by calculating the two formulas as below [23–25]:

$$\theta_{R(n)} = \theta_{R(0)} - [\theta_{R(0)} - S]\left(\frac{n}{\theta_N}\right)^{\lambda}, \tag{3}$$

$$\beta_{R(n)} = \frac{\ln[\ln 2]}{\ln\left[\frac{R_{\{50\}}(0) - [R_{\{50\}}(0) - S](n/N_{\{50\}})^{\lambda}}{\theta_{R(0)} - [\theta_{R(0)} - S](n/\theta_N)^{\lambda}}\right]}, \tag{4}$$

where $R_{\{50\}}(0) = \theta_{R(0)}(\ln 2)^{1/\beta_{R(0)}}$, $N_{\{50\}}(0) = \theta_N(\ln 2)^{1/\beta_N}$, $\theta_{R(0)}$ and θ_N are the scale parameters of static strength and fatigue life, respectively; while $\beta_{R(0)}$ and β_N are the shape parameters of static strength and fatigue life, respectively.

3.2. Fatigue reliability of two-level amplitude loading

When a composite laminate is subjected to a two-level loading, the residual strength is affected by the accumulating effect of fatigue failure. Assuming at stress level S_1, the fatigue life of composite laminate specimens is N_1; and at stress level S_2, the fatigue life of composite laminate specimens is N_2, the corresponding degradation parameters of residual strength are respectively λ_1 and λ_2. From equation (1) we can obtain the degradation curves for residual strength at constant amplitude loadings S_1 and S_2 as

$$R_1(n_1) = R(0) - [R(0) - S_1]\left(\frac{n_1}{N_1}\right)^{\lambda_1}, \tag{5}$$

$$R_2(n_2) = R(0) - [R(0) - S_2]\left(\frac{n_2}{N_2}\right)^{\lambda_2}. \tag{6}$$

If a composite laminate specimen is subjected to stress S_1 and has passed n_1 fatigue cycles, and is then subjected to stress S_2 for n_2 fatigue cycles, the residual strength can be shown to be

$$R_2(n_{12} + n_2) = R(0) - [R(0) - S_2]\left(\frac{n_{12} + n_2}{N_2}\right)^{\lambda_2}. \tag{7}$$

When the instantaneous stress changes from S_1 to S_2, the two residual strengths of the composite laminate should be the same value, namely

$$R_1(n_1) = R_2(n_{12}), \tag{8}$$

so the equivalent fatigue cyclic period n_{12} can be calculated from equations (5) and (6) as

$$n_{12} = \left[\frac{R(0) - R_1(n_1)}{R(0) - S_2}\right]^{1/\lambda_2} N_2. \tag{9}$$

If the composite laminate was fractured in the second period under stress S_2, then the fatigue reliability function is

$$\mathcal{R}_{\mathbf{R}_2(n_{12}+n)}[S_2] = \mathcal{P}[\mathbf{R}_2(n_{12}+n) > S_2] = \exp\left[-\left(\frac{S_2}{\theta_{R_2(n_{12}+n)}}\right)^{\beta_{R_2(n_{12}+n)}}\right]. \quad (10)$$

3.3. Fatigue reliability of multiple-level amplitude loading

The condition of composite materials that are subjected to multi-level amplitude loading is the extension of the former condition in this study. If the composite laminate was subjected to k levels of stress $S_1, S_2, \ldots$ and S_k step by step, and passed through the corresponding $n_1, n_2, \ldots$ and n_k cyclic periods, the degradation curve for residual strength can be written as

$$R_k(n_{(k-1)k} + n_k) = R(0) - [R(0) - S_k]\left(\frac{n_{(k-1)k} + n_k}{N_k}\right)^{\lambda_k}, \quad (11)$$

where

$$n_{(k-1)k} = \left[\frac{R(0) - R_{k-1}(n_{(k-2)(k-1)} + n_{k-1})}{R(0) - S_k}\right]^{1/\lambda_k} N_k. \quad (12)$$

In this condition, the fatigue reliability function can be obtained by

$$\mathcal{R}_{\mathbf{R}_k(n_{(k-1)k}+n)}[S_k] = \exp\left[-\left(\frac{S_k}{\theta_{R_k(n_{(k-1)k}+n)}}\right)^{\beta_{R_k(n_{(k-1)k}+n)}}\right]. \quad (13)$$

3.4. Fatigue reliability of random loading

According to the above method of fatigue failure accumulation, the Monte Carlo simulation method can obtain the residual strength of composite laminates in one cyclic period, and then evaluate the fatigue reliability. The procedures are:

(a) Input the necessary material constants and θ, β, which are related to the random variables $R(0)$, S.

(b) Generate the random value of static strength $R(0)$.

(c) Generate the random value of fatigue stress S, and calculate the fatigue life N and the degradation parameter λ of residual strength.

(d) Calculate the degradation of residual strength at this period $\Delta R(n)$. The calculating method is

$$\Delta R(n) = -[R(0) - S]\lambda \frac{n^{\lambda-1}}{N^{\lambda}}. \quad (14)$$

(e) Calculate the residual strength $R(n)$ by

$$R(n) = R(n-1) + \Delta R(n). \quad (15)$$

(f) Accumulate the fatigue cyclic period and repeat Step (c) to Step (e) until the pre-set fatigue cycle number n is reached; then we can determine the laminate residual strength.

In different cyclic period conditions $n_1, n_2, n_3, \ldots,$ this simulation method was repeated several times to yield a series of residual strength value. Analyzing the simulation results will determine the distribution function of residual strength. Using the stress–strength interference theory, we can estimate the fatigue reliability of composite laminates for every cyclic period.

4. RESULTS AND DISCUSSION

4.1. Experimental results for static strength

Ten Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminate specimens were tested by the tensile test to find the static strengths, which are also the residual strengths $R(0)$ in the condition of zero cycle, as shown in Table 1. To study the reliability of composite laminates, the static strengths $R(0)$ of these laminates are fitted well by the Weibull distribution function. Then we can plot the values of data on the Weibull distribution probability paper, as shown in Fig. 3, thus the scale parameter and shape parameter can be found. The two parameters of distribution function are also shown in Table 1.

4.2. Experimental results for fatigue life

It is well known that fiber reinforced composites have a fairly high longitudinal tensile strength and stiffness. In fact, the Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates possess very high specific strength and stiffness in our experiments. The fatigue life of the specimens would reach 1 000 000 cycles when the fatigue stress, which

Table 1.
Static strength of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates

Static strength $R(0)$ (MPa)	723.13
	725.19
	743.20
	748.85
	754.77
	770.98
	774.84
	776.90
	788.99
	796.70
Mean value $\mu_{R(0)}$	760.36
Standard deviation $\sigma_{R(0)}$	25.42
Scale parameter $\theta_{R(0)}$	771.73
Shape parameter $\beta_{R(0)}$	34.06

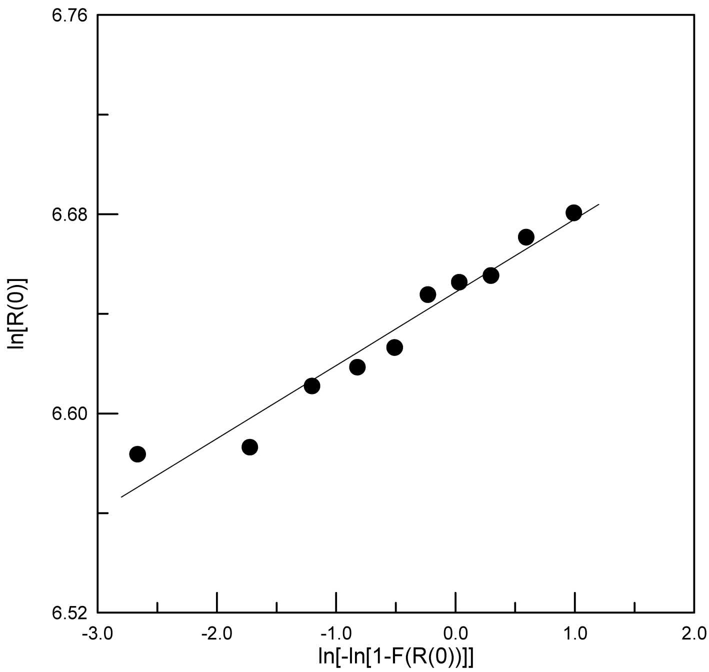

Figure 3. Weibull distribution probability paper of static strength of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates.

was the 72% of the static strength R(0), was applied. It will be recognized that the life of specimens is infinity. To implement the experimental work efficiently under appropriate conditions, we chose three different percentages of mean static strength $\overline{R(0)}$, namely 85%, 82% and 79% to be the maximum test fatigue stresses S_1, S_2 and S_3, where

$$S_1 = 85\% \, \overline{R(0)} = 646.31 \text{ MPa}, \tag{16}$$

$$S_2 = 82\% \, \overline{R(0)} = 623.50 \text{ MPa}, \tag{17}$$

$$S_3 = 79\% \, \overline{R(0)} = 600.68 \text{ MPa}. \tag{18}$$

We used five composite laminates to carry out the fatigue tests for the three stress levels, respectively. Then we obtained the fatigue life of constant amplitude loading as shown in Table 2, and determined the formula of relationship for fatigue life and fatigue stress as

$$\frac{S}{\overline{R(0)}} = -0.028 \ln \overline{N} + 1.099. \tag{19}$$

The fatigue life N of the composite laminates is fitted well by the Weibull distribution function. By plotting the values of data on the Weibull distribution probability paper, as shown in Fig. 4, the cumulative distribution function parameter θ_N, β_N can be found. The values were also shown in Table 2.

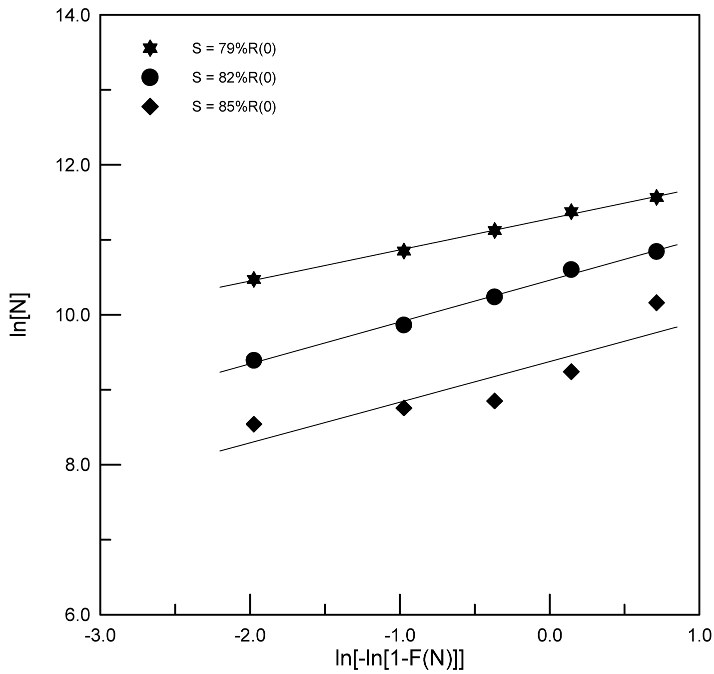

Figure 4. Weibull distribution probability paper of fatigue life of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates under constant amplitude loading.

Table 2.
Fatigue life of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates under constant amplitude loading

	Fatigue life N (cycles)		
	S_1	S_2	S_3
	5109	11966	35312
	6350	19191	51444
	6978	27895	67774
	10288	40184	87143
	25864	51185	105347
Mean value μ_N	10918	30084	69404
Standard deviation σ_N	8572	15803	27812
Scale parameter θ_N	11791	34895	79313
Shape parameter β_N	1.84	1.79	2.40

4.3. Estimation of reliability and experimental results for constant amplitude loading

We prepared other Gr/PEEK composite laminates for the fatigue tests under the different stress levels S_1, S_2 and S_3 until the fatigue cyclic periods n were reached

Table 3.
Residual strength of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates under constant amplitude loading S_2

Residual strength $R(n)$ (MPa)			
$R(20\%\overline{N})$	$R(40\%\overline{N})$	$R(60\%\overline{N})$	$R(80\%\overline{N})$
710.28	700.34	(11751)*	(12649)*
722.85	707.71	(15820)*	(13984)*
730.78	716.53	672.53	(16024)*
742.64	728.30	684.05	(18425)*
749.55	738.44	701.81	(22643)*
757.60	748.91	709.43	628.16
766.21	750.65	719.92	635.32
769.99	756.89	724.27	659.35
774.38	764.76	730.55	668.69
785.07	777.59	736.93	674.46
		748.65	681.54
		758.12	689.49
			695.98
			708.08
			726.14

at 20%, 40%, 60% and 80% of the fatigue life $\overline{N}$ for each stress level, respectively. Then the composite laminates underwent the tensile tests and the residual strengths $R(n)$ under the different cycles were obtained as shown in Table 3 and Fig. 5 (considering the text length, we have just presented the condition for S_2). In Table 3, the "*" mark represents that those composite laminates are fractured before the testing condition is reached, and the numbers in the brackets are the fatigue cycles when the laminates are fractured. From Fig. 5, we can find the residual strengths in mono degradation tendency. The degradation parameters λ for each stress level are $\lambda_1 = 1.648$, $\lambda_2 = 1.685$ and $\lambda_3 = 1.730$, respectively. Through them, we can obtain the formula of relationship for λ and S as

$$\frac{S}{R(0)} = -0.728\lambda + 2.049. \tag{20}$$

Upon examining these results, we found that the residual strength $R(0)$ of composite laminates are fitted well by the Weibull distribution function, so we can plot the values of data on the Weibull probability papers, as shown in Fig. 6. Using equations (3) and (4), we can determine the distribution parameters of residual strength $R(n)$, $\theta_{R(n)}$ and $\beta_{R(n)}$ under the arbitrary fatigue cyclic period. Then we employed these two parameters to estimate the fatigue reliability of composites using equation (2). Figure 7 is the comparison between theory estimations of fatigue reliability and experimental results. As can be seen, they agree closely.

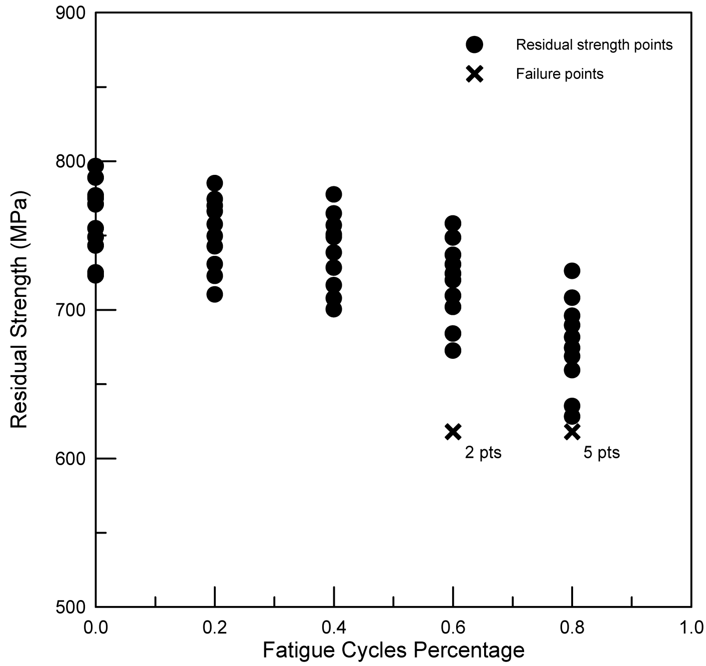

Figure 5. Residual strength of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under constant amplitude loading S_2.

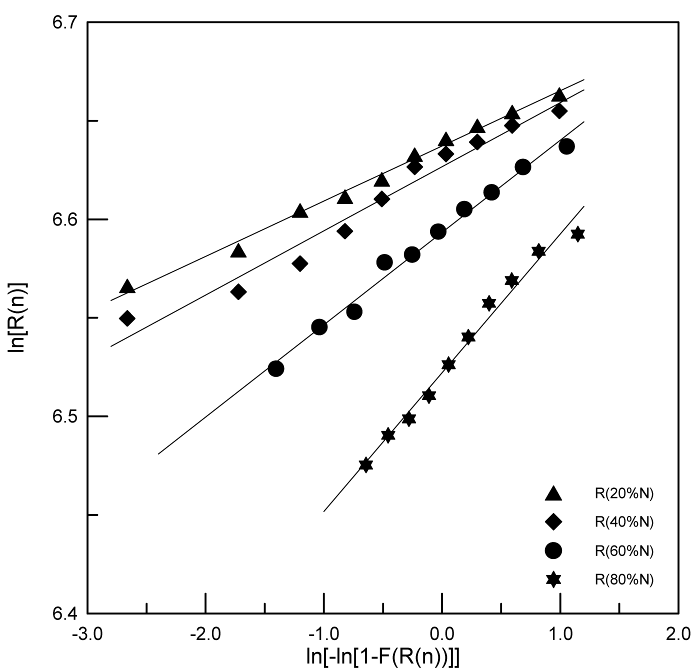

Figure 6. Weibull distribution probability paper of residual strength of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under constant amplitude loading S_2.

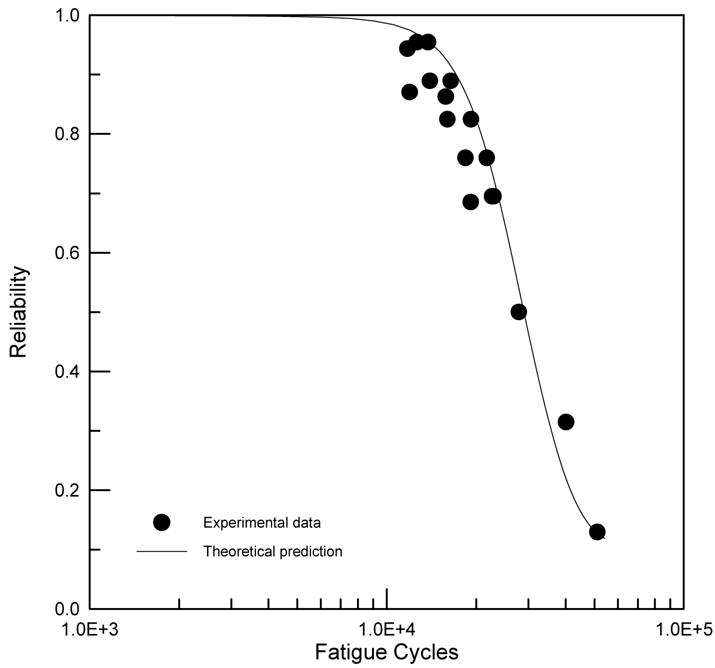

Figure 7. Reliability of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under constant amplitude loading S_2.

4.4. Estimation of reliability and experimental results for multiple-level amplitude loading

In the two-level loading experiment, the specimens were subjected to the first-level fatigue stress until the fatigue cycle reached 20% of mean fatigue life. They were then subjected to the second-level fatigue stress until they were fractured, and the fatigue cycle values were recorded. Four kinds of stress condition were chosen for fatigue tests as below. Ten composite laminate specimens were used in each condition, respectively, to repeat the experiment, and the results are shown in Table 4.

(a) first-level stress S_1, second-level stress S_3;

(b) first-level stress S_1, second-level stress S_2;

(c) first-level stress S_3, second-level stress S_1;

(d) first-level stress S_3, second-level stress S_2.

In another three-level loading experiment, the specimens were subjected to the first-level fatigue stress until the fatigue cycle reached 20% of mean fatigue life; then they were subjected to the second-level fatigue stress until the fatigue cycle reached 40% of mean fatigue life; and finally they were subjected to the third-level fatigue stress until they were fractured, and the fatigue cycle values were recorded. Two kinds of stress condition were chosen for fatigue tests as below. Ten

Table 4.
Fatigue life of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under two-level loading

Fatigue life N (cycles)			
S_1–S_3	S_1–S_2	S_3–S_1	S_3–S_2
29436	8050	3760	15348
36974	11766	5190	20329
41591	16667	6266	21353
46468	20436	8195	23390
51042	22087	10504	26033
53160	23713	12899	26242
55467	28999	13671	28111
58999	31045	15131	32439
64126	38938	17517	33970
73805	43211	21492	40015

Table 5.
Fatigue life of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under three-level loading

Fatigue life N (cycles)	
S_1–S_3–S_2	S_3–S_1–S_2
1988	7393
7923	11585
12528	15897
14616	18139
15518	20605
16725	23018
18113	26706
21496	28848
24890	33659
32659	40280

composite laminate specimens were used in each condition, respectively, to repeat the experiment, and the results are shown in Table 5.

(a) first-level stress S_1, second-level S_3, third-level S_2;

(b) first-level stress S_3, second-level S_1, third-level S_2.

The distribution parameters $\theta_{R(n)}$ and $\beta_{R(n)}$ of residual strength $R(n)$ for the composites under multiple-level loading can be calculated from equations (3) and (4), and the results were substituted into equation (10) or (13) to estimate the fatigue reliability of composites. Figures 8 and 9 show the comparisons between theory estimations of fatigue reliability and experimental results.

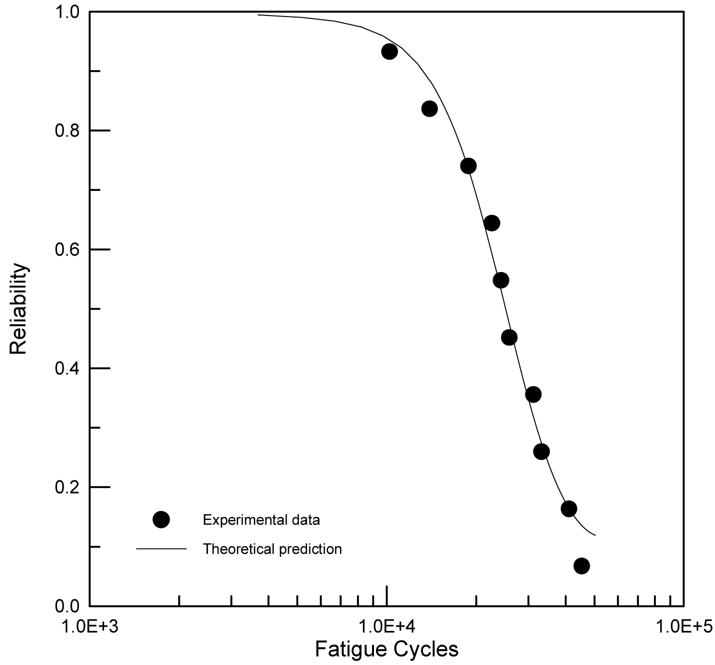

Figure 8. Reliability of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under two-level amplitude loading S_1–S_2.

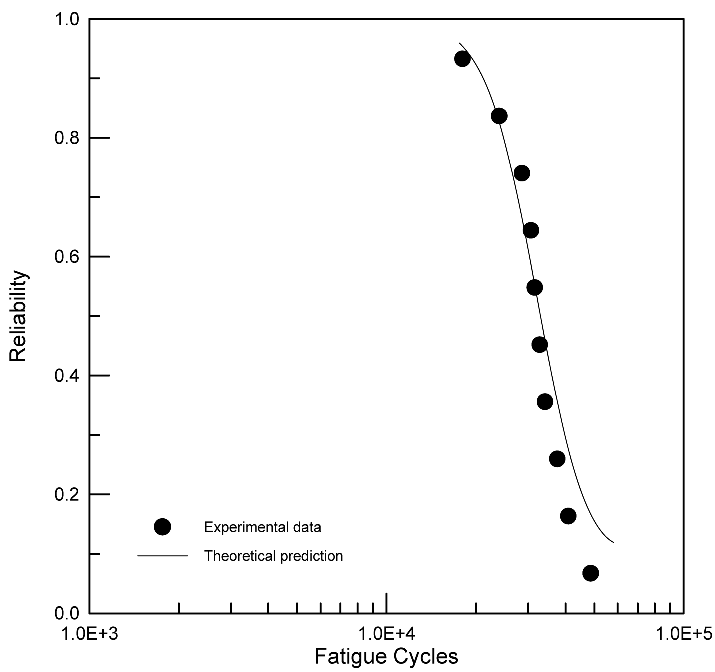

Figure 9. Reliability of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under three-level amplitude loading S_1–S_3–S_2.

4.5. Estimation of reliability and experimental results for random loading

Three types of random loading S_{p1}, S_{p2}, S_{p3} were designed for the fatigue experiment, the distribution functions are shown in equations (21), (22) and (23):

$$\mathcal{F}_{\mathbf{S}_{p1}}[S_{p1}] = 1 - \exp\left[-\left(\frac{S_{p1} - 577.87}{\theta_{S_{p1}}}\right)^2\right], \tag{21}$$

$$\mathcal{F}_{\mathbf{S}_{p2}}[S_{p2}] = 1 - \exp\left[-\left(\frac{S_{p2} - 577.87}{\theta_{S_{p2}}}\right)^2\right], \tag{22}$$

$$\mathcal{F}_{\mathbf{S}_{p3}}[S_{p3}] = 1 - \exp\left[-\left(\frac{S_{p3} - 577.87}{\theta_{S_{p3}}}\right)^2\right], \tag{23}$$

where $\theta_{S_{p1}} = 68.44$, $\theta_{S_{p2}} = 45.63$ and $\theta_{S_{p3}} = 22.81$.

We used the BASIC language program to control the MTS810 material testing machine for generating random stresses. Ten composite laminate specimens were used in each type, respectively, to repeat the experiment, and the results are shown in Table 6.

Furthermore, we used the Monte Carlo simulation method to calculate the residual strength of composites, and determine the fatigue reliability of composites. Figure 10 displays the comparisons between simulations of fatigue reliability and experimental results.

5. CONCLUSIONS

This study conducted tensile tests and constant amplitude loading, two-level loading, three-level loading and random loading fatigue tests on Gr/PEEK [0/45/90/-45]$_{2S}$ composite laminate specimens using the residual strengths of composites and the theory of probability and statistics. We were able to develop a probability analy-

Table 6.
Fatigue life of Gr/PEEK [0/45/90/-45]$_{2S}$ laminates under random loading

Fatigue life N (cycles)		
S_{p1}	S_{p2}	S_{p3}
802	4661	6129
2012	6995	9762
4308	9857	23741
7341	11656	47421
10028	27974	65438
14604	34135	101852
17611	52762	126646
30107	69032	145303
44093	94480	204948
73282	146855	383965

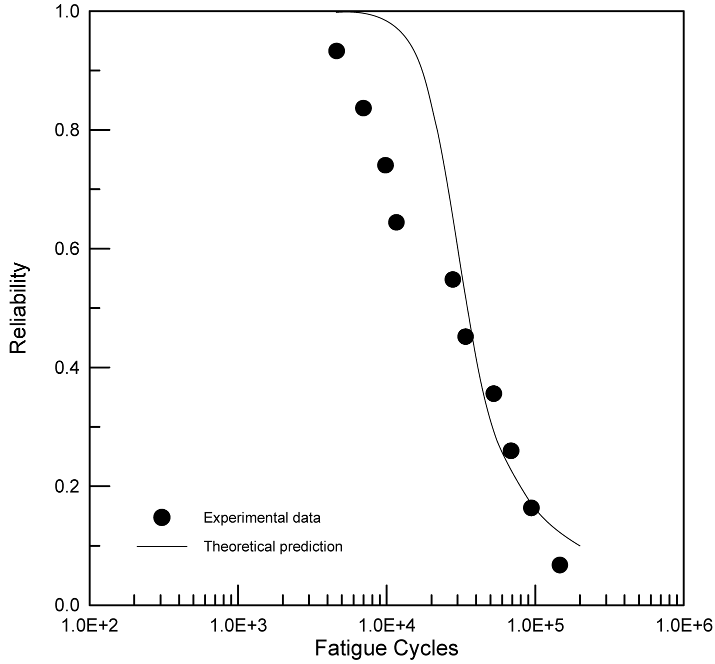

Figure 10. Reliability of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ laminates under random loading.

sis model of Gr/PEEK composites for different loading conditions, and then predict and estimate the fatigue probability of composite laminates.

Under constant amplitude loading, the residual strength of Gr/PEEK $[0/45/90/\text{-}45]_{2S}$ composite laminates will decay when the fatigue cycles increase. In the present study, we can properly describe the residual strength degradation by equation (1). Moreover, the residual strength of composite laminates is fitted well by the Weibull distribution function. This conclusion is in agreement with most of the researches that have discussed the fatigue phenomena of composite materials.

Under multiple-level loading, the present study also uses the same residual strength degradation to establish the probability model of fatigue damage accumulation, and then determines the fatigue reliability of composite materials under different fatigue stress levels. Under random loading, we adopted the Monte Carlo simulation method to estimate the fatigue reliability of composite materials. According to the analysis results under different loading conditions, the estimated models of fatigue reliability that we developed can estimate the fatigue reliability of Gr/PEEK composite material with a high degree of accuracy.

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